Work Sheet (7)
(Vectors and Translation)

Question:

Given the four points A, B, C, and D such that:

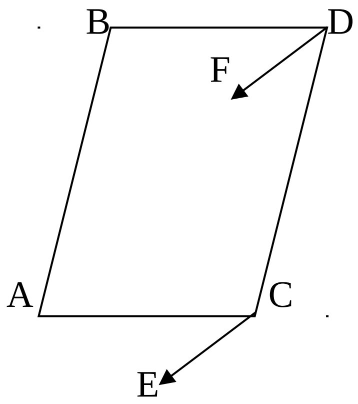
$\overrightarrow{AB} = \overrightarrow{CD}$ and the points E and F such that:

$\overline{DF} = \overrightarrow{CE}$

Show that:

a) $\overrightarrow{AE} = \overrightarrow{BF}$

b) $\overrightarrow{AB} = \overrightarrow{EF}$



Question:

VSOP and VDQS are two parallelograms with common side [VS].

Show that: $\overrightarrow{PD} = \overrightarrow{PQ}$

Question:

G is the centroid of a triangle ABC and M is the mid point of [AB].

a) Show that: $\overrightarrow{GA} + \overrightarrow{GB} = 2\overrightarrow{GM}$

b) Deduce that: $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$

Question:

Consider 3 non – collinear points A, B, and C.

a) Construct the points D, E, and F such that:

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$, $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{AD}$, $\overrightarrow{AF} = \overrightarrow{AE} + \overrightarrow{AC}$

b) Show that D is the mid point of [CE] and [AF].

Question:

Given a circle of center O and diameter [AB]. Let M be a point on the circle, N its image in the translation of vector $\overrightarrow{AB}$, and P the symmetric of N with respect to B.

a) What is the nature of each of the quadrilaterals AMNB and AMBP? Deduce that P belongs to the circle.
b) Show that triangle MNP is isosceles.
c) Let G be the point of intersection of (NO) and (BM). Show that (PG) bisects [MN].

Question:

Let ABCD be a parallelogram.

a) Construct the point E, image of D in the translation of vector $\overrightarrow{AD}$.
b) Construct the point F, image of B in the translation of vector $\overrightarrow{AB}$.
c) Show that: $\overrightarrow{EC} = \overrightarrow{CF}$

Question:

Given any four points A, B, C, and D of the plane.

a) Prove that: $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{AD} + \overrightarrow{CB}$ and $\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AD} + \overrightarrow{BC}$
b) Simplify: $\vec{U} = \overrightarrow{AC} + \overrightarrow{BD} + \overrightarrow{CB}$ and $\vec{V} = \overrightarrow{AC} + \overrightarrow{BD} + \overrightarrow{CB} - \overrightarrow{DA}$

Question:

Given the 3 points: $A(-2,7)$, $B(5,3)$, and $C(-3,-1)$

a) Plot the points A, B, and c.
b) Determine the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$, and deduce the length of [AB] and [AC].
c) Construct the point D, image of C in the translation of vector $\overrightarrow{AB}$.
d) What is the nature of the quadrilateral ABDC? Justify/
e) Calculate the coordinates of point D.

<u>Question:</u>

Given the 3 points: A (- 3, 1), $B(5,7)$, and $C(4,0)$

a) Prove that the triangle ABC is right isosceles.
b) Find the coordinates of I, the circumcenter of triangle ABC. Calculate the radius of this circle.
c) Let (D) be the tangent to the circumcircle at C. Find the equation of (D).
i) Find the coordinates of A', B', and C', the respective images of A, B, and C in the above translation.
ii) Find the equation of (D'), the image of (D) in the above translation.

<u>Question:</u>

Given the 3 points: A (5, 0), B (-3, 0), and C (1, 4).

a) Construct the 3 points D, E, and F such that: $\overrightarrow{AD} = \overrightarrow{CB}$, E is the image of A in the translation of vector $\overrightarrow{BC}$, and F is the symmetric of A with respect to D.
b) Find the coordinates of D, E, and F.
c) Show that ADBC is a square, and BCEF is an isosceles trapezoid.
d) Show that: $\overrightarrow{EF} = 3 \times \overrightarrow{CB}$
e) Find the coordinates of I, the point of intersection of (FC) and (EB).

<u>Question:</u>

Consider a triangle ABC with M the mid point of [BC].

a) Construct the points D and E such that $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$ and $\overrightarrow{AE} = \overrightarrow{AC} - \overrightarrow{AB}$
b) Compare $\overrightarrow{DC}$ and $\overrightarrow{CE}$. What do you conclude?
c) Let I be the mid point of [BM] and J the mid point of [CM].
d) Prove that: $\overrightarrow{AI} + \overrightarrow{AJ} - \overrightarrow{AB} + \overrightarrow{AC}$.

Chapter Test

Question 1:

Let ABCD be a rhombus with center O.

1) Determine $\overrightarrow{BC} + \overrightarrow{BA}$ and $\overrightarrow{AB} + \overrightarrow{AD}$.
2) Determine $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$

Question 2:

Draw a triangle ABC and the medians [BJ] and [CI]. Let D be the symmetric of B with respect to J, and E the symmetric of C with respect to I.

1) Find $\overrightarrow{CB} + \overrightarrow{CA}$ and $\overrightarrow{BA} + \overrightarrow{BC}$
2) Compare $\overrightarrow{AE}$ and $\overrightarrow{AD}$
3) Deduce that A, D, and E are collinear and that A is the mid point of [DE].

Question 3:

Consider the point A (3, 2) and the vector $\vec{V}(2,-1)$

Determine the coordinates of the points B and C such that:

$\overrightarrow{AB} = \overrightarrow{CA} = \vec{V}$

Question 4:

Consider an orthonormal system of axes x`Ox, y`Oy

1) Locate the points A (2,1), B (2, 2) and C (1, 2) and construct the point D, image of A in the translation of vector $\overrightarrow{BC}$.
2) What is the nature of quadrilateral ABCD? Justify.
3) Find the components of $\overrightarrow{BC}$, then deduce the coordinates of D.

Question 5:

Consider the points A (3, 3), B (2, -2), and C (-2,4).

1) What is the nature of triangle ABC? Justify.
2) Let E be the image of C by the translation of vector $\overrightarrow{AB}$. Determine the coordinates of $\overrightarrow{CE}$. Deduce the coordinates of E.
3) Let (S) be the circle circumscribed about triangle ABC and denote by I its center.
   a) Find the coordinates of I and calculate the radius of (S).
   b) Determine the coordinates of the points of intersection of (S) with $y'y$

Question 6:

Consider the 4 points: A (-2, -3), B (5, -5), C (3,4), and D (-4, 6).

a) Find a translation that takes [AD] to [BC].
b) Find the equation of line (AD) as well as the equation of its translation in the above translation.

Work Sheet (8)
(Trigonometry)

Question:

Let x be an acute angle such that: sin x = $\frac{3}{7}$. Calculate cos x and tan x.

Question:

Consider a triangle ABC right angled at A. Use a calculator to complete the following table, by angles correct to the nearest 0.01°, and by segments correct to the nearest 0.01 cm.

$\hat{B}$	$\hat{C}$	BC	AB	AC
63°		8 cm		
	39°		18 cm	
			5 cm	12 cm

Question:

Given the 4 points A (6, 1), B (1, -4), C (-3, 2), and D (4, -1).

a) Calculate the slopes of (AB) and (CD),

b) Find the measure of the angle that (AB) makes with the positive x-axis, as well as the angle that (CD) makes with the positive x-axis.

Question:

Given the two points A(1, 2), B(-1, 3).

a) Find the equation of the straight line (D) passing through A and making an angle of 30° with the positive x-axis.

b) Find the equation of the straight line (D') passing through B and making an angle of 120° with the positive x-axis.

Question:

Prove the following relations:

a) $(\cos x + \sin x)^2 = 1 + 2\sin x \cdot \cos x$

b) $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$

c) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

d) $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

e) $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

f) $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = \dfrac{2}{\sin x}$

Chapter Test

Question 1:

Prove the following relations:

a) $\left(1+\tan^2 x\right)\cos^2 x = 1$.

b) $(\cos x - \sin x)^2 = 1 - 2\sin x \cdot \cos x$.

c) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$.

d) $\sin^4 x - \cos^4 x = (\sin x - \cos x)(\sin x + \cos x)$

Question 2:

Show that: $(x\sin a - y\cos a)^2 + (x\cos a + y\sin a)^2 = x^2 + y^2$.

Question 3:

Given a triangle ABC such that: AB = 4cm, $\hat{B} = 40^\circ$, and $\hat{C} = 35^\circ$, and let [AH] be the height relative to [BC].

a) Calculate AH, BH, CH, and AC correct to the nearest 0.01 cm.

b) Calculate the perimeter and area of triangle ABC.

Question 4:

Consider the points A (-2, 2), B (1, 3) and C (4,-2) (orthonormal system).

a) Determine the coordinates of the point D such that $\overrightarrow{CD} = \overrightarrow{AB}$.

b) Find the components of $\overrightarrow{AD}$ and those of $\overrightarrow{BC}$.

c) Find the equation of the straight line (AB) as well as the measure of the angle that it makes with the x-axis.

Question 5:

Let RST be a triangle right at R, and let [RH] be the height relative to [ST]. Calculate the sides and the angles of triangle RST knowing that: RH = 7cm, SH = 5cm, and RT = 9cm.

Question 6:

Let ABC be a triangle with heights [AD], [BE], [CF] and orthocenter H.

a) Find two fractions for $\cos \hat{A}$.

b) Deduce that the triangles AEF and ABC are similar.

Question 7:

A circle has a diameter AB = 10cm. Let [AP] be a chord such that: $B\hat{A}P = 41^\circ$. The tangent to the circle at B meets the line (AP) in T. Calculate BT, AT, and TP.

Question 8:

Draw a circle of O and radius R= 4cm. From a point A outside the circle, the tangents [AP] and [AQ] to the circle are drawn such that: $P\hat{A}Q = 38^\circ$. Calculate AP and PQ.

Work Sheet (9)
(Geometry: Revision Exercises And Problems)

Question:
Let ABCD be parallelogram with center O. Let M and N be the mid points of [OB] and [OD] respectively. Show that ANCM is a parallelogram.

Question:
Let ABCD be a parallelogram. Let E be the symmetric of A with respect to B, and F the symmetric of A with respect to D. Show that:

a) BDFC and BDCE are parallelograms.
b) The points E, C, and F are collinear.
c) C is the mid point of [EF].

Question:
Let ABC be a triangle, M the mid point of [BC], O the mid point of [AM], D the mid point of intersection of (BO) and (AC), and E the point of intersection of (AC) with the line drawn from M parallel to (BD). Show that: AD = DE = EC.

Question:
Let ABCD be a quadrilateral with perpendicular diagonals. MNPQ is the quadrilateral formed by joining the mid points of the sides of ABCD. Show that MNPQ is a rectangle.

Question:
ABC is a triangle such that (AB < AC). The bisector of $\hat{A}$ cuts [BC] at D. The perpendicular from B to [AD] cuts [AD] at M and cuts [AC] at E. N is the mid – point of [BC]. Prove that $MN = \frac{1}{2}(AC - AB)$.

Question:
ABC is an isosceles triangle with vertex A. The bisector of $\hat{B}$ meets [AC] at D. The perpendicular through D to [BD] cuts [BC],
(produced if necessary) at E. M is the mid-point of [BE]. Prove that:

a) $A\hat{D}B = 3A\hat{B}D$.
b) The triangle DMC is isosceles.

Question:

ABC is a triangle in which $\hat{A} = 90^\circ$. Draw the height [AH] and the median [AM]. [HE] is drawn perpendicular to [AB] and [HF] is drawn perpendicular to [AC]. Prove that:

a) EF = AH.
b) $A\hat{E}F = \hat{C}$ and $H\hat{E}F = \hat{B}$
c) Deduce that [EF] and [AM] are perpendicular.

Question:

Let (AB) and (CD) be two parallel lines cuts by a transversal at E and F respectively. The bisectors of $A\hat{E}F$ and $B\hat{E}F$ cuts (CD) at M and N respectively. The bisector of $C\hat{F}E$ cuts [ME] at S, and the bisector of $D\hat{F}E$ cuts [NE] at T. Show that:

a) ETFS is a rectangle.
b) (ST) is a parallel to (AB).

Question:

Let ABCD be a parallelogram, and M the mid point of [AB]. The parallel to [MD], drawn through B, cuts [DC] at N. Shown that N is the midpoint of [DC].

Question:

Let ABC be a triangle such that: $\hat{B} = 40^\circ$ and $\hat{C} = 60^\circ$. The bisector of $\hat{A}$ cuts [BC] at D. Let H be the foot of the perpendicular drawn from D to [AB].

a) Prove that the triangle DAB is isosceles.
b) Let E be the symmetric of D with respect to H. Prove that ADBE is a rhombus.

Question:

Let ABCD be a trapezoid such that: $AB = AD = \frac{1}{2}DC$ and [AD] $\perp$ [BC].

Let E the midpoint of [DC] and M the midpoint of [AD]. The parallel to [DB], drawn from E, cuts [BC] at I. Show that:

a) ABED is a rhombus.
b) NBIE is a rectangle (N is the center of the rhombus ABED).
c) M, N, and I are collinear.

Question:

Consider a triangle ABC, right-angled at A. Let [AH] be the height relative to [BC]. Let D and E be the symmetries of H with respect to [AB] and [AC] respectively.

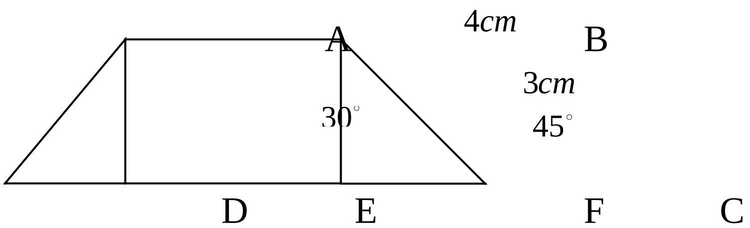
a) Show that the triangle ABH and ABD are congruent, as well as triangle AHC and AEC.
b) Deduce that the points D, A, and E are collinear, and that BDEC is a right trapezoid.
c) Show that the triangle DHE is right.
d) Let AC = b and AB = c. Calculate, in terms of b and c, the area of triangle ABC and deduce that of the trapezoid BDEC.

Question:

Let ABCD be a square of side 5cm. The semi-straight line [CX] is drawn exterior to the square and such that $\widehat{DCX} = 15^\circ$. The perpendicular to [Ac] at A cuts [CX] at E. Calculate AE and CE.

Question:

Calculate the perimeter and the area of the adjacent trapezoid.



Question:

Let ABC be an isosceles triangle such that AB = AC = 2cm and $\widehat{BAC} = 120^\circ$. Let M be the mid point of [BC]. The perpendicular to [AB] at B cuts (AM) at D. Calculate AM, BM, BC, AD, and BD.

Question:

Let ABC be a right triangle such that: $\hat{A} = 90^\circ$, $\hat{B} = 30^\circ$, and AC =5ccm. Let M be the mid point of [BC]. The perpendicular to [BC] at M cuts (AC) at N.

a) Calculate AB, MN, and NC.
b) Show that AMBN is an isosceles trapezoid and calculate its perimeter.

Question:

Let ABCD be a trapezoid with bases AB = 6cm and CD = 9cm and such that: $\hat{A} = \hat{D} = 90^\circ$ and $A\hat{C}D = 30^\circ$

a) Calculate AD, AC, and BC.
b) The perpendicular to [AC] at A cuts (CD) at E. Show that the quadrilateral ABCE is an isosceles trapezoid.

Question:

Let ABC be an equilateral triangle of side 6cm, and (D) the line drawn through A perpendicular to [AC]. Let H and L be the feet of the perpendiculars drawn from B to (AC) and (D) respectively.

a) Calculate BH and the area of triangle ABC.
b) Let I be the point on [LA] such that LI = 3cm, and E the point of intersection of (CB) and (D). Let F be the point on (D) such that AF = AC, (A is between L and F).
   i)Show that the triangle BLI is right isosceles and calculate BI.
   ii)Calculate the angles of the triangle ABF and BCF.
   iii)Show that the triangles EBI and ECF are similar.

c) Let T be the foot of the perpendicular to (AB) drawn from L. Calculate LT and TA.

Question:

Let ABC be an isosceles triangle ABC with apex A inscribed in a circle. The bisectors of $\hat{B}$ and $\hat{C}$ cut the circle at E and F respectively. Let I be the point of intersection of (BE) and (CF).

a) Show that AF = BF = AE = EC.
b) Show that $F\hat{A}E = F\hat{I}E$ and $A\hat{E}I = A\hat{F}I$
c) Deduce that the quadrilateral AFIE is a rhombus.

Question:

Given a circle of center O, radius 2cm, and diameter [AB]. D is a point on the circle such that AD =2cm. Through D, a line parallel to (AB) is drawn to cut the circle at C.

a) Show that the quadrilateral ABCD is an isosceles trapezoid.
b) Show that the triangle ABC is semi-equilateral.
c) Let I the point of intersection of (AD) and (BC). Find $A\hat{I}B$.
d) Let E be the point of intersection of (AC) and (BD). Find $A\hat{E}B$.
e) Show that (EI) is the perpendicular bisector of [AB].
f) (CD) and the tangent at B to the circle intersect at F. Calculate BF and DF.

Question:

Let (c) be a circle of center O and radius R. Mark any point M on the circumference of (c). Draw another circle (C') of center M and radius R. (C') intersects with (c) at A and B. Draw through A a common secant EAD.

a) Show that the quadrilateral AOBM is a rhombus. Calculate its angles.
b) Show that the triangle BED is equilateral.

Question:

Draw a circle of center O. Let [AB] and [DE] be two perpendicular diameters and let C be any point on AE.

a) Show that [CD] bisects $A\hat{C}D$.
b) Let H be the foot of the perpendicular drawn from B to [CD]. Shown that the triangle CHB is right isosceles.
c) (BH) cuts the circle at N. Show that (DN) is parallel to (CB).
d) (HO) cuts (CB) at M. Show that (HM) is the perpendicular bisector of [CB].

Question:

Let ABC be a triangle inscribed in a circle. The height [AH] when produced cuts the circumference at M and the height [BH'] when produced cuts the circumference at N. [BH'] and [AH] meet at K. Show that:

a) [AC] is the bisector of $N\hat{A}M$.
b) [HH'] and [MN] are parallel.

Question:

Let (O) and (O') be two circles tangent internally at A. Through a point T on the common tangent at A, a tangent to circle (O) is drawn touching it at M, and a tangent to circle (O') is drawn touching it at N. Show that the triangle TMN is isosceles.

Question:

Let [AB] be a diameter of a circle whose radius is R. A point C is marked on the tangent at A such that AC = 2R. (BC) cuts the circle at D. Show that:

a) $A\hat{D}B = 90^\circ$
b) AD = CD.

Question:

Consider a semi-circle of center O and diameter [AB]. M is any point on the semi-circle. The tangent at A and M intersect at T. Let P and Q be the feet of the perpendiculars drawn from M to (AB) and (TA) respectively. Let I be the mid-point of [PQ]. Show that:

a) $A\hat{I}O = 90^\circ$
b) The points O, I, and T are collinear.
c) [MA] is the bisector of each of the angles: $O\hat{M}Q$ and $P\hat{M}T$.

Question:

Let ABC be an equilateral triangle inscribed in a circle of center O and radius R. (BO) cuts the circle at K. The perpendicular to [AC] at A cuts (CK) at M.

a) Show that the triangles ABK and ACM are congruent.
b) Show that the triangle AKM is equilateral.
c) Show that the quadrilateral ABOM is a parallelogram.
d) Calculate, in terms of R, the area of ABOM.

Question:

Let (O) and (O') be two circles tangent externally at P. [APB] and [DPC] are two common perpendicular secants drawn through P. Draw (TPT') the common tangent through P. Show that:

a) [AD] and [BC] are parallel.
b) $(AD + BC)^2 = AB^2 + DC^2$

Question:

A right-angled triangle ABC is inscribed in a circle of center O and radius R in such a way that AB = 2R and AC = R.

a) Find the measure of each angle of the triangle ABC.

b) [BA] is produce by a length AD = R. The bisector of $C\hat{A}B$ cuts (OC) at H and cuts (BC) at M. Show that (DC) is tangent to the circle at C that it is parallel to (AM).
c) Find, in term of R, the length of DC, AH and AM.